Inconsistencies and Errors

Foundations for Numeracy: An Evidence-based Toolkit for Early Learning Practitioners

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Abstract:

In 2010, the Canadian Child Care Federation released the book "Foundations of Numeracy" which is aimed at early childhood educators/workers and elementary school level teachers. The problems with this document is that they don't just focus on correct definitions, consistent knowledge in mathematics and correct data in mathematics. The authors are focusing more on concepts than the actual numeracy which educators should be aware of before applying the techniques they propose. In this paper, I would also like to suggest better practices in the propositions I shall give for possible experimentation based on the correct data.

0.0 Introduction:

"Foundations for Numeracy" by the collaboration of the Canadian Child Care Federation and the Canadian Language and Literacy Research Network is an ineffective perspective on Mathematics for multiple reasons. They focus too heavily on numeric techniques and not enough on introductory mathematics. The focus is on numbers rather than the simpler objects in mathematics based on the actual basis of mathematics I have observed heavily throughout early level developments until secondary school level in mathematics curriculum and styles of learning within Canada and other countries such as the United States of America. This paper has two goals:

1. Pointing out the flaws of "Foundations of Numeracy", and incorrect information presented.
2. Introducing simpler, lower level mathematical objects could possibly provide a better grasp of mathematics in later years versus arithmetic which is a little more complicated than set theoretic approaches. These concepts would provide a cleaner, simpler path to reaching arithmetic. This includes a better understanding of what mathematics is and how it could be useful in connecting ideas. It could also provide a better introduction to logical ideas and the drawing of lines between solving problems and solving them effectively.

In this paper I will be focusing on the mathematical topics and pointing them out section by section in italic headers. I will be using the same headers as the authors' in review of each respectively.

1.0 Critique

Before I begin discussing the sections given I would like to make some remarks in particular to the assumptions of the document. They assume quite an array of factors which probably could be changed for efficiency.

1.1 Part 1: Cognitive Processing: Information Processing

"Foundations of Numeracy" assumes addition is the simplest operation for the brain of a child to use which is most likely is not true (FoN 8). In early years of child development, visual cognition is one of the strongest developments (the ability to observe visual instruction) in the brain.

The mind in early years of a child is still in it's early stages of development in abstract faculties but, visual instructions are great impressions on a child.. For machines, addition is one of the simplest instructions outside of set theory and logical construction (for example, the traditional Adder of a CPU on a Computer) but, this is not a human being. Even for the computer, there exist simpler operations. Human beings do not operate on binary machine code. Addition for a human may require more than just the memory of understanding the instruction, applying the mechanism instruction, calling on cognitive traits that map onto the instructions, and then operating on them with the reflection at each step of the brain. This is rather complicated versus trying to approach more visually powerful tools of learning since there are better mathematical objects out there for that than addition or arithmetic. Aiming more easy to visualize tools that are mathematically simpler to approach can improve the ways of approaching useful problems versus arithmetic which is limited to only number systems which are incomplete. The arithmetic of basic counting systems like numbers are far easier to grasp once simpler objects are understood since they pave the building roots to understanding how to use basic mathematics. I will discuss those in Section 2.0.

1.2 Part 1: Types of Knowledge

The following examples given are horrible examples of the types of knowledge used by the brain. To begin let us look at the factual knowledge example they have given us:

> “factual knowledge is information that can be learnt by memorization and repetition (i.e. rote learning), such as knowing that 2+2=4...” (FoN 9)

Response:

2 +2 does not always equal 4. That is a blind assertion to make and is a common misconception since very few education systems emphasize that only works within a closed system. Let us show why this doesn't hold...

Assume 2+2 = 4. Let the symbol 2 be defined as the number 3, and the symbol 4 be the number 2. Now if 2+2 always equaled 4 then 4 should equal 2 + 2 by the condition that if 4 then 2+2, likewise if 2+2 then, 4 (if and only if). The following modification made the to systems of integers does contain all the integers, and is reflexive, and properly is defined. To make this clear let us show what happens:

Let there exist a Turing Machine T1 which can add two integers (which can exist), and T2 which can decompose two integers into symmetric values (which is what we want based on the above). We will test both system A's and system B's statements for validity. If 2 + 2 always equals 4 holds, then anything closed under addition should work with T1 with the elements {0,2,4}. If 4 always equals 2 + 2 then anything closed under addition should work with T2 with the elements {0,2,4}. We will use T1, and T2 to decide the validity.

Let elements a,b be positive integers closed in the system A (holding to the rules of integers).

Let the system B have the following rules:

I) 4:= 2

II) 2:= 4

The following operations were described above in the description. Now let us assume that 2+2 always equals 4.

Begin with T1 tests:

With system A, we try a+a = b, where a:= 2, b:=4 as an assertion. T1 returns the tape with the correct output of 4. With system B, we will try the same assertion. T2 returns an invalid tape output (it will outcome with 8 not 4). So the universal statement of if 2+2, then 4 is false.

Now with T2 tests:

With system A, we try a = b+b, where a:=4, b:=2 as an assertion. T2 returns the tape with the correct output. With system B we try the same assertion. T2 will return an invalid output since it will return an output similar to 1+1 and not 2+2. So the universal statement of if 4 then, 2+2 is false.

Therefore, it is invalid to assume 2+2 = 4 for all numeric systems by contradiction. ■

Conclusion of Response:

Increasing the awareness of systems of numbers would be a good idea if one would like to teach them. Such concepts would be the number line, and basic sets of numbers (naturals, integers,...). Numbers typically are not a good choice for examples of universal statements from since arithmetic is incomplete (Godel). I would recommend much better examples than blind and false assertions.

“procedural knowledge... For example, knowing how to solve the problem 2 + 3 by continuing to count “3,4,5,...”” (FoN 9).

Response: The following is an example of factual knowledge rather than procedural knowledge. What is being described is an algorithm. The mental cognition of the algorithm itself is a procedural cognition but knowing how to do the algorithm itself is factual knowledge. The ability to count to 3,4,5 is the procedural component to the procedural knowledge but, the described action is something memorized by doing an algorithm which is as mathematically memorized as what I described in the previous response. Because counting is a cognition based on factual knowledge, the action of carrying out this algorithm is the procedural knowledge component. For instance, how does a computer know to move to the next line in an instruction? It is designated after reading a line to fetch the instruction fed to figure out what to do next. This could be a branch to another routine, or even just moving to the next line. The execution of this is the procedural component versus the rules laid out which is the factual component.

Conclusion of Response:

Find a better example of this. I would recommend a child counting from 1 to 10 with guidance without reminder of the numbers 1 to 10 except the next number.

To close off this part, I would recommend the next edition of this book get a professional with a mathematical background to check the correctness of the information presented. There are many generalizations made involving the operations of arithmetic, mathematical knowledge and numerous other concepts. This will come up more than once in this book as we progress into definitions.

1.3 Part 1: Math Anxiety

I wish to comment on this section as a whole since this section makes a vast variety of assertions about Mathematics. It begins by asserting that situations dealing with numbers cause stressful situations (FoN 11). I think that this is an assertion based on the current situation with the education of mathematics and not mathematics in general. The term “Math Anxiety” is relative, based on the current situation in the education of mathematics. If the ordering of the topics taught were changed introducing simpler topics before the others originally in place, I am sure our topic of “Math anxiety” would deplete. This phenomenon is obvious to someone with a mathematics background, if it was explained more formally. Addition and other arithmetic operations are tricky, and require a jump above from nothing. By injecting simpler and more useful concepts in before arithmetic, it can provide a jump that makes naturally more sense to early ages in the development of children. I will mention these propositions briefly in Section 2.0. The current system of education in 'maths' (I emphasize the quotations since most education systems do not teach mathematics even at secondary school level) have a poorly structured way of introducing concepts to children. Numbers and arithmetic can be a staggering concept to a child unless they are introduces to connecting ideas at an earlier stage. I strongly believe that this anxiety would be minimized if the ordering of materials is given much considerable change over all years of childhood education. The emphasis on arithmetic over the years is a very poor way to begin mathematical knowledge of the world. I will suggest a proposed idea in Section 2.0.

As a side note I'd like to mention is that the definition of an algorithm is incorrect. An algorithm in the formal sciences is a finite set of steps to effectively solve a problem. The emphasis is finite, the book doesn't state finite which is incredibly important. Undecidable problems can have algorithms if that were the case since not stating finite lets the infinite into the picture which is not an accurate reflection of an algorithm.

1.4 Part 1: Transfer of Learning

One thing I wanted to mention on this section was that most of the concepts presented in current educational systems are not that 'transferable'. That is, there is no logical underroot to the problems in these situations. No transfer of information tools is given as they are just isolated and not connected by any rules that a child could pass along from problem to problem.

1.5 Part 2: Counting Procedures

I wanted to point out this section due to it's incorrect use of definitions. One to one correspondence is one of them. The example given is a horrible one. It's contradictory to the definition. You can assign an element to a symbol and likewise to another symbol. Most computer programs do this all the time, and most mathematics do as well. For instance, I can assign 5 to be the value 3, and the value 3 to also be 3. In a computer program I could assign the variable x to be a pointer to some address in memory then potentially reuse it for another purpose. We could also let x be two values in sync with each other without any contradictory nature involved. I think the author was trying to focus on matching of items but, used a very poor example to describe it. I could assign an item in a set to be “3” and also to “5” like so:

Suppose element x, and set X. Whenever I see x I can represent it as “3” and “5”. So when I see X = {3,5,2} = {3,2} = {5,2} this works well since under the hood it's still X = {x,x,2} = {x,2}. I would recommend the author finds a better example.

One huge problem I have with this document is that it never defines which numbers the author is trying to convey. I could be working in a system where “1,2,4” is in order, versus “1,2,3,4”. More explicit mention of number systems would be very helpful.

1.6 The remaining components of Section 2

I thought the remaining components were quite vague of what materials they entail and I encountered more incorrect definitions. There was no focus on actual mathematical concepts. The authors of this book seem to focus completely on numbers and very little on actual problem solving for children at their early years. There is very little room in this book for possible jumps to better techniques such as introducing other concepts to allow more effective problem solving skills such as introducing more engaging activities relating to logic since that's where the heart of mathematical reasoning is. Without this core component early construction of mathematics to children, this becomes much harder due to lacking reason to actually use the tools they are taught.

2.0 Propositions

In this section I would like to suggest some propositions for this organization to look into for possible future research into childhood research. I find from years of tutoring students and working with children that the patterns they seem to be reflecting on in this document are of these results. I would like to see them improved significantly by focusing on a completely different end than traditional 'math' education (once again I want to emphasize that this isn't mathematics since the current system does not teach mathematics). Here are some that I propose. They are all related to a common idea. That set theory may be the best approach to begin with:

- *Less pocus on numbers:* Numbers in the real world are all around us but, they bear no meaning unless there is an associated value and meaning to them. That is what other mathematical objects are for.
- *Focus more on Simpler Operations:* I would highly recommend teaching more useful conventions and working naturally towards arithmetic from these conventions. A wonderful example of this is set theory. The operations of sets or even multi-sets are much simpler than arithmetic and can be depicted in much more visual ways than numbers and basic arithmetic can. For example:

  Which is simpler?

  Suppose we have two bins of dolls. To add them is most costly than doing a union of the bins. One can count the dolls quicker since they don't need to be in any class of anything since addition requires the bins to have other properties that make the dolls 'dolls' versus just being an item of some criteria.
- *Working with sets:* Much like the previous point, set theoretic operations like union and intersection are actually simpler for a child to grasp since they are much easier to visually interact with and simpler to express. It works as a natural growing and exploring ground to work towards arithmetic since it permits either taking the root of directly discovering arithmetic from set theory or having a stepping stone towards arithmetic. The operation of a union and addition to a child are very similar for instance.
- *Incorporates logic*: The wonderful fact about sets are their ease of relating to logic.

This is very useful when extending ideas and connecting ideas for children and solving problems. The means that is currently in place forces a child to think with zero tools and in later years never gets them until they are an adult providing they get formal education on the subject which is easily a trigger for the anxiety and hatred towards mathematics present in reports. Even slowly incorporating this over years would be a wonderful way to begin critical thinking faculties in the development of the brain.

3.0 Conclusion

In conclusion, I would like to see these propositions given in section 2.0 experimented with. We are moving into the 21st century, and the emphasis on mathematical knowledge is growing rapidly. A means of encouraging mathematical thought by making it far more interactive and easier to grasp in pieces is the best approach to reaching this goal. Eventually, it would be wonderful to see children learning and exploring in vast new ways that were never used in earlier generations. The focus on critical thinking should be brought in at an early age and would provide a new tone in mathematical education and actually let children and students begin thinking more for themselves and ask questions to their educators and parents. In this learning more and taking advantage of the mathematical rules of science whether if it may be the formal or empirical sciences.

All sources (FoN) are from Foundations of Numeracy (2010). Canadian Child Care Federation.